ELEC387 — Power electronics

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1 Power electronics systems `pp. 3-15`

Main task: process and control flow of electric energy by supplying voltage and current in a form that is optimally suited for user loads.
In the year 2000: 50% of the electrical load supplied through power electronics.
Linear electronics: low efficiency because transistor are used as resistors.
Switch mode: efficient because fully off or fully on.
@ high frequency: transformers and filters are small.
Selection of f_s: compromise between switching power dissipation (increases with f_s) and cost of transformer (decreases with f_s).
Outputs (2): dc (regulated or adjustable) and ac (constant frequency, adjustable magnitude or adjustable frequency and magnitude).
Categories (4): ac to dc, dc to ac, dc to dc, ac to ac.
Converter: generic term.
Rectifier: Power from ac to dc.
Inverter: Power from dc to ac.
Line frequency converter: naturally commutated (50 or 60Hz).
Switching converter: $f_s >>$.
Resonant and quasi-resonant converter: switching @ $v = 0$ and/or $i = 0$.

2 Overview of power semiconductor switches `pp. 16-32`

Diodes: on and off states controlled by the power circuit. Reverse-recovery time t_{rr} required to block negative polarity voltage. SCHOTTKY, fast-recovery, line-frequency.
Thyristors: on state controlled by gate pulse, off state controlled by the power circuit. Turn-off time interval t_q during which a reverse voltage must be applied. Phase-control thyristors, inverter-grade thyristors, light-activated thyristors.
Ideal switches: block arbitrarily large forward and reverse voltages with zero current flow when off, conduct arbitrarily large currents with zero voltage drop when on, switch from on to off and vice versa instantaneously, vanishingly small power required to trigger the switch.

$$P_T = \underbrace{\frac{1}{2} V_d I_o f_s (t_{c(\text{on})} + t_{c(\text{off})})}_{P_s} + \underbrace{V_{\text{on}} I_o \frac{t_{\text{on}}}{T_s}}_{P_{\text{on}}}$$

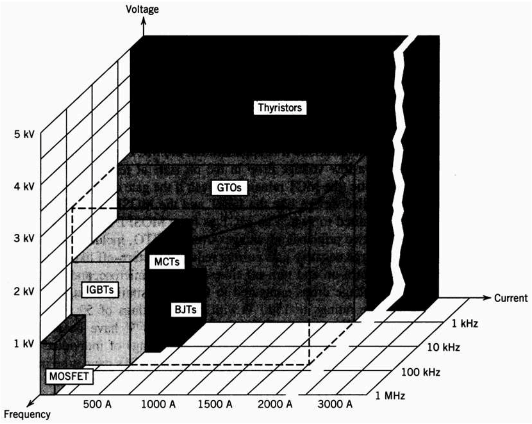

Figure 1: Overview of devices capabilities

3 Review of basic electrical and magnetic circuit concepts `pp. 33-60`

Steady-state: reached when the circuit waveforms repeat with a time period T.

$$\%\mathrm{THD}_i = 100\frac{I_{\mathrm{dis}}}{I_{s1}} = 100\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100\sqrt{\sum_{h\neq 1}\left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

$$\text{Crest factor} = \frac{I_{s,\mathrm{peak}}}{I_s}$$

$$\mathrm{PF} = \frac{I_{s1}}{I_s}\cos\phi_1 = \frac{I_{s1}}{I_s}\mathrm{DPF} = \frac{1}{\sqrt{1+\mathrm{THD}_i^2}}\mathrm{DPF}$$

Inductor: current cannot change instantaneously, net change of flux over T is zero, areas in volt-second must be equal.
Capacitor: current cannot change instantaneously, net change of charge over T is zero, areas in ampere-second must be equal.

4 Computer simulation of power electronic converters and systems `pp. 61-76`

Modeling and computer simulations play an important role in the analysis, design, and education of power electronic systems. It is important to simplify the system being simulated to be consistent with simulation objectives.

5 Line-frequency diode rectifiers: line-frequency ac → uncontrolled dc `pp. 79-120`

Power flow: only from ac side to dc side.
Capacitor: large to avoid ripple, filter effect.

5.1 Single-phase diode bridge rectifier `pp. 82-100`

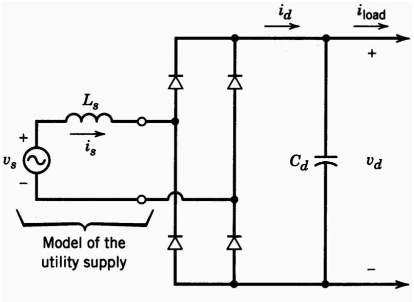

Figure 2: Single-phase diode bridge rectifier

Idealized circuit with $L_s = 0$: two groups of diodes (D_1, D_2) and (D_3, D_4), transition between the two groups is instantaneous due to $L_s = 0$. Average value obtained by integrating v_s over one-half time period

$$V_{do} = \frac{2}{\pi}\sqrt{2}V_s \simeq 0.9V_s \quad \%\text{THD}_i \simeq 48.43\% \quad \text{DPF} = 1.0 \quad \text{PF} = 0.9$$

Effect of L_s on current commutation: transistion of the ac-side current i_s from a value of $+I_d$ to $-I_d$ will not be instantaneous, commutation interval u, all four diode conduct during the commutation interval ($v_d = 0$). The current through inductor L_s changes from $-I_d$ to $+I_d$.

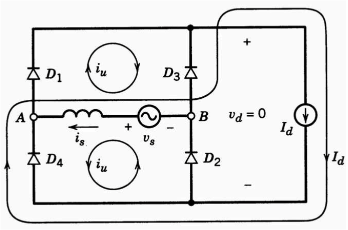

Figure 3: Single-phase diode bridge rectifier during commutation

$$A_u = \int_0^u \sqrt{2}V_s \sin\omega t \mathrm{d}(\omega t) = \omega L_s \int_{-I_d}^{I_d} \mathrm{d}i_s = 2\omega L_s I_d = \sqrt{2}V_s(1 - \cos u)$$

$$\cos u = 1 - \frac{2\omega L_s}{\sqrt{2}V_s} I_d \quad V_d = 0.9V_s - \frac{2\omega L_s I_d}{\pi}$$

Constant dc-side voltage $v_d(t) = V_d$: approximation with a large value of C. When v_s exceeds V_d, diodes 1 and 2 begin to conduct. The average value I_d depends on the value of V_d, I_d decreases when V_d increases because $|v_s| - V_d$ decreases.
Rectifier characteristic: For a given value of I_d, increasing L_s results in a smaller $I_{\text{short circuit}}$. Increasing L_s results in improved i_s waveform with a lower THD$_i$, a better PF and a lower crest factor.

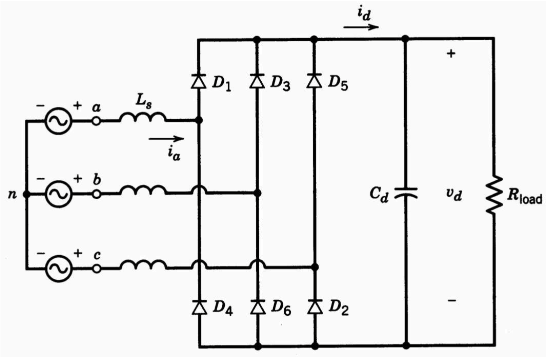

Figure 4: Three-phase diode bridge rectifier

5.2 Three-phase, full-bridge rectifiers `pp. 103-112`

Better: lower ripple and higher power handling capability.
Idealized circuit with $L_s = 0$: six segments per cycle of line frequency (six-pulse rectifier), each diode conducts for 120°, commutation instantaneous. Average value obtained by integrating v_{ab} over a 60° interval.

$$V_{do} = \frac{3}{\pi}\sqrt{2}V_{LL} \simeq 1.35V_{LL} \quad \text{DPF} = 1.0 \quad \text{PF} = \frac{3}{\pi} \simeq 0.955$$

Effect of L_s on current commutation: during the commutation between phase c and a v_{Pn} remains at v_{cn}

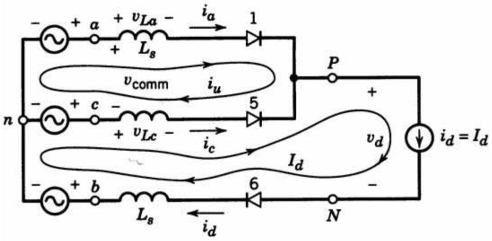

Figure 5: Three-phase diode bridge rectifier during commutation

$$\cos u = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_{LL}} \quad V_d = 1.35V_{LL} - \frac{3}{\pi}\omega L_s I_d$$

Constant dc-side voltage $v_d(t) = V_d$: similar to the single-phase case.
Rectifier characteristic: better THD$_i$, PF and DPF. THD$_i$ decreases when L_s increases.
Comparison of single-phase and three-phase rectifiers: single-phase rectifier contains significantly more distortion, this results in a much poorer PF. DPF is high in both rectifier. Ripple is smaller in the three-phase rectifier thus the capacitor can be smaller. It is always preferable to use a three-phase rectifier.

6 Line-frequency phase-controlled rectifiers and inverters: line-frequency ac ↔ controlled dc `pp. 121-160`

Power flow: from ac to dc and vice versa. Inverter mode of operation on a sustained basis is possible only if a source of power is present on the dc side.

6.1 Single-phase converters `pp. 126-138`

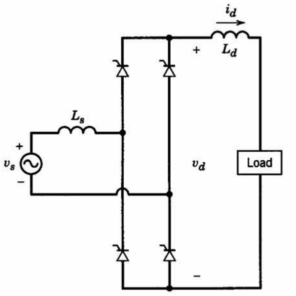

Figure 6: Single-phase thyristor converter

Idealized circuit with $L_s = 0$ and $i_d(t) = I_d$: commutation is instantaneous. Average value obtained by integrating from α to $\pi + \alpha$, becomes negative beyond $\alpha = 90°$

$$V_{d\alpha} = \frac{2\sqrt{2}}{\pi} V_s \cos\alpha \simeq 0.9 V_s \cos\alpha \quad P \simeq 0.9 V_s I_d \cos\alpha$$

Line current i_s: square wave with an amplitude of I_d shifted by the delay angle α

$$\%\mathrm{THD}_i = 48.43\%$$

Power:

$$\mathrm{DPF} = \cos\alpha \quad \mathrm{PF} = 0.9\cos\alpha$$

Effect of L_s: current commutation takes a finite commutation interval u, during which all four thyristors conduct ($v_d = 0$, $v_{Ls} = v_s$). The reduction in volt-radian area due to the commutation interval is the integral of v_{L_s} from α to $\alpha + u$.

$$\cos(\alpha + u) = \cos\alpha - \frac{2\omega L_s I_d}{\sqrt{2} V_s} \quad V_d = 0.9 V_s \cos\alpha - \frac{2}{\pi}\omega L_s I_d$$

Input line current i_s: essentially a trapezoidal waveform

$$\mathrm{DPF} \simeq \cos(\alpha + \frac{1}{2}u)$$

Discontinuous-current conduction: occurs at light loads or beyond a certain value of E_d because $|v_s| - E_d$ decreases. At constant α, if I_d falls below a threshold that depends on α, V_d increases sharply because the conduction is now discontinuous and V_d remains at the value E_d for a much longer time.
Inverter mode of operation: possible only if there is a source of energy on the dc side, $90° < \alpha < 180°$. Linear relation between V_d and I_d if i_d is constant (large L_d). Knowing E_d, we can determine I_d and hence the power flow P_d.

$$E_d = V_d = V_{do}\cos\alpha - \frac{2}{\pi}\omega L_s I_d$$

For start-up, α is made sufficiently large (e.g., 165°) so that i_d is discontinuous, then α is decreased to obtain the desired I_d and P_d.

6.2 Three-phase converters `pp. 138-153`

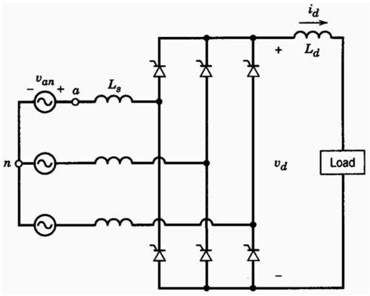

Figure 7: Three-phase thyristor converter

Idealized circuit with $L_s = 0$ and $i_d(t) = I_d$: commutation is instantaneous. Average value obtained by determining the volt-radian area A_α

$$V_{d\alpha} = V_{do} - \frac{A_\alpha}{\pi/3} \simeq 1.35 V_{LL} \cos\alpha = V_{do}\cos\alpha \quad P \simeq 1.35 V_{LL} I_d \cos\alpha$$

Input line current i_a, i_b and i_c: rectangular waveforms with an amplitude I_d shifted by the delay angle α

$$\%\mathrm{THD}_i = 31.08\%$$

Better than the single-phase.
Power:

$$\mathrm{DPF} = \cos\alpha \quad \mathrm{PF} = \frac{3}{\pi}\cos\alpha \simeq 0.955\cos\alpha$$

Better than the single-phase.
Effect of L_s: the German VDE standards require that Z_s must be a minimum of 5%. Current commutation take a finite commutation interval u. The reduction in volt-radian area due to the commutation interval is the integral of v_{L_s} from α to $\alpha + u$.

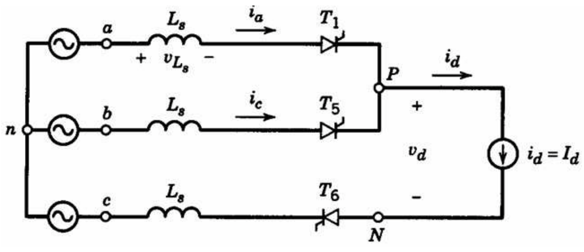

Figure 8: Three-phase thyristor converter during commutation

$$\cos(\alpha + u) = \cos\alpha - \frac{2\omega L_s}{\sqrt{2}V_{LL}} I_d \quad V_d = \frac{3\sqrt{2}}{\pi} V_{LL}\cos\alpha - \frac{3\omega L_s}{\pi} I_d$$

Input line current i_s: the waveform can be approximated to be trapezoidal

$$\text{DPF} \simeq \cos(\alpha + \frac{1}{2}u)$$

The ac-side inductance L_s reduces the magnitudes of the harmonic currents. A greater inductances leads to a greater u

Discontinuous-current conduction: occurs at light loads or beyond a certain value of E_d because $|v_s| - E_d$ decreases. At constant α, if I_d falls below a threshold that depends on α, V_d increases sharply because the conduction is now discontinuous and V_d remains at the value E_d for a much longer time.

Inverter mode of operation: possible only if there is a source of energy on the dc side, $90° < \alpha < 180°$. Linear relation between V_d and I_d if i_d is constant (large L_d). Knowing E_d, we can determine I_d and hence the power flow P_d. For start-up, α is made sufficiently large (e.g., 165°) so that i_d is discontinuous, then α is decreased to obtain the desired I_d and P_d.

Line notching and distortion: There are six commutations per line-frequency. During each commutation, two out of three phase voltages are shorted together by the converter thyristors through L_s in each phase. A line-to-line voltage is shorted twice per cycle resulting in deep notches (ringing due to capacitances can also occur).

$$v_a - v_b - (v_A - v_B) = L_s \left(\frac{\mathrm{d}i_a}{\mathrm{d}t} - \frac{\mathrm{d}i_b}{\mathrm{d}t} \right)$$

$$\Rightarrow \omega L_s(\Delta i_a - \Delta i_b) = 2\omega L_s I_d = \text{Deep notch area } A_n$$

$$\text{Deep notch area depth} \simeq \sqrt{2} V_{LL} \sin\alpha$$

$$\text{Notch width } u \simeq \frac{2\omega L_s I_d}{\sqrt{2} V_{LL} \sin\alpha}$$

$$\text{Voltage \%THD} = 100 \frac{\left[\sum_{h \neq 1} (I_h \omega L_{s1})^2 \right]^{\frac{1}{2}}}{V_{\text{phase(fundamental)}}}$$

7 dc-dc switch mode converters `pp. 161-199`

Task: convert unregulated dc input into a controlled dc output at a desired voltage level.

Converters: only the step-down and the step-up are the basic converter topologies. All others are derived from those basic topologies.

Modes: continuous and discontinuous modes of operation.

Drawbacks of basic converter (2): in practice the load would be inductive thus the switch would have to absorb the inductive energy, the output voltage fluctuates between 0 and V_d

7.1 Step-down (buck) converter `pp. 164-172`

Output voltage: the average output voltage is obtained by equating the volt-second areas across the indutor. It varies linearly with D.

$$V_o = \frac{t_{\text{on}}}{T_s} V_d = D V_d$$

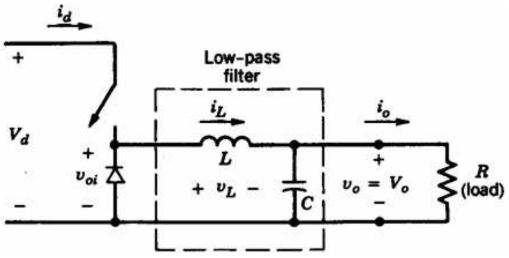

Figure 9: Step-down (buck) converter

Continuous-conduction mode: the inductor current flow continuously. During t_{on}, $v_L = V_d - V_o$ and during t_{off}, $v_L = -V_o$. Neglecting power losses associated with all the circuit elements, the input power P_d equals the output power P_o. **Boundary between continuous and discontinuous conduction:** the inductor current goes to zero at the end of the off period.

$$I_{LB} = \frac{1}{2} i_{L,\text{peak}} = \frac{t_{\text{on}}}{2L}(V_d - V_o) = \frac{DT_s}{2L}(V_d - V_o) = \frac{T_s V_d}{2L} D(1-D) = \frac{T_s V_o}{2L}(1-D)$$

Discontinuous-conduction mode with constant V_d: in application such as a dc motor speed control.

$$I_{LB,\text{max}} = \frac{T_s V_d}{8L} \quad \text{when} \quad D = 0.5$$

$$I_{LB} = 4 I_{LB,\text{max}} D(1-D)$$

Occurs when the output load power is decreased thus the average inductor current will decrease. This dictates a higher V_o

$$(V_d - V_o)DT_s + (-V_o)\Delta_1 T_s = 0 \Rightarrow \frac{V_o}{V_d} = \frac{D}{D + \Delta_1}$$

$$i_{L,\text{peak}} = \frac{V_o}{L}\Delta_1 T_s \Rightarrow I_o = i_{L,\text{peak}} \frac{D + \Delta_1}{2}$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{1}{4}(I_o / I_{LB,\text{max}})}$$

Discontinuous-conduction mode with constant V_o: in application such as dc power supplies.

$$I_{LB,\text{max}} = \frac{T_s V_o}{2L} \quad \text{when} \quad D = 0$$

$D = 0$ is hypothetical because it would lead to $V_d = +\infty$

$$I_{LB} = I_{LB,\text{max}}(1-D)$$

$$D = \frac{V_o}{V_d}\left(\frac{I_o / I_{LB,\text{max}}}{1 - V_o / V_d}\right)^2$$

Output voltage ripple: all of the ripple component in i_L flows through the capacitor and its average component flow through the load resistor. The ripple is independent of the output load power so long as the conduction mode is continuous.

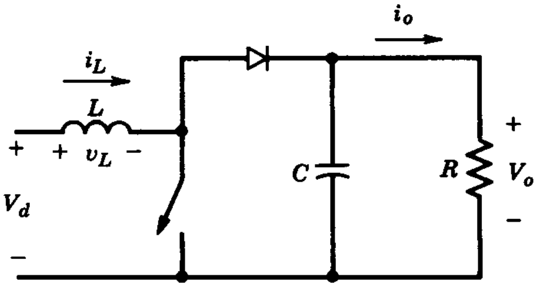

Figure 10: Step-up (boost) converter

7.2 Step-up (boost) converter `pp. 172-178`

Continuous-conduction mode: the inductor current flows continuously

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

Boundary between continuous and discontinuous conduction: the inductor current goes to zero at the end of the off period.

$$I_{LB} = \frac{1}{2} i_{L,\text{peak}} = \frac{1}{2}\frac{V_d}{L} t_{\text{on}} = \frac{T_s V_o}{2L} D(1-D)$$

$$I_{LB} = 4D(1-D) I_{LB,\text{max}} \quad I_{oB} = \frac{27}{4} D(1-D)^2 I_{oB,\text{max}}$$

Discontinuous-conduction mode: only with V_0 kept constant (power supplies). When I_o is below a certain threshold, D must be varied in order to keep the same conversion ratio.

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,\text{max}}}\right]^{\frac{1}{2}}$$

Effect of parasitic elements: due to losses associated with the inductor, the capacitor, the switch, and the diode. Poor switch utilization at high value values of D.

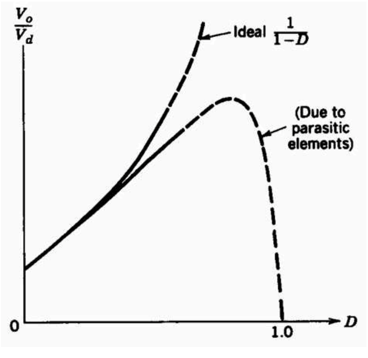

Figure 11: Effect of parasitic elements on voltage conversion ratio

7.3 Buck–boost converter `pp. 178-184`

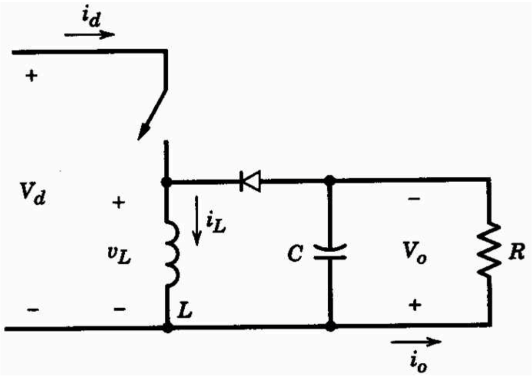

Figure 12: Buck–boost converter

Applications: where a negative-polarity output may be desired.
Continuous-conduction mode: equating the integral of the inductor voltage over one time period to zero yields

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

Boundary between continuous and discontinuous conduction: the inductor current goes to zero at the end of the off period.

$$I_{LB} = I_{LB,\max}(1-D) \quad I_{oB} = I_{oB,\max}(1-D)^2$$

Discontinuous-conduction mode: only with V_0 kept constant (power supplies). When I_o is below a certain threshold, D must be varied in order to keep the same conversion ratio.

$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oB,\max}}}$$

7.4 Cúk dc-dc converter `pp. 184-188`

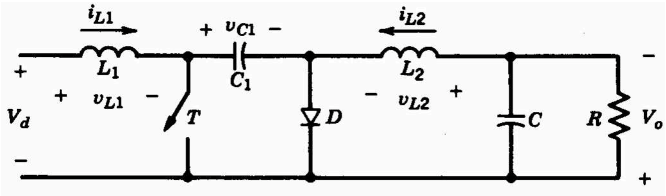

Figure 13: Cúk converter

Applications: where a negative-polarity output may be desired. C_1 acts as the primary means of storing and transferring energy.

$$V_{C_1} = V_d + V_o \quad \frac{V_o}{V_d} = \frac{D}{1-D}$$

Advantage: input and output currents reasonably ripple free.
Disadvantage: requirement of a capacitor C_1 with a large ripple-current-carrying capability.

7.5 Full-bridge dc–dc converter `pp. 188-194`

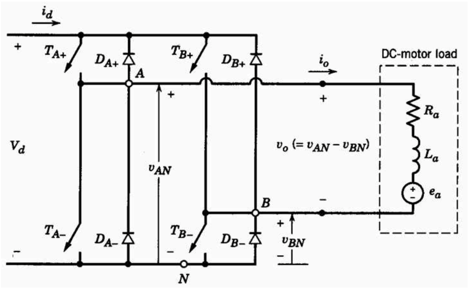

Figure 14: Full-bridge converter

Applications (3): dc motor drives, dc-to-ac (sine-wave) conversion in single-phase uninterruptible ac power supplies, dc-to-ac (high intermediate frequency) conversion in switch-mode transformer-isolated dc power supplies. Operates in all four quadrants of the i_o–v_o plane.
Antiparallel diodes: difference between the on state and conducting state of the switch. When a switch is turned on, it may or may not conduct depending on the direction of i_o.
Leg: two switches and their antiparallel diodes. When one switch is on the other one is off and vice versa.

$$v_{AN} = V_d \quad \text{(if } T_{A+} \text{ is on and } T_{A-} \text{ is off)}$$

$$v_{AN} = 0 \quad \text{(if } T_{A-} \text{ is on and } T_{A+} \text{ is off)}$$

$$V_{AN} = V_d \cdot \text{duty ratio of } T_{A+} \quad V_{BN} = V_d \cdot \text{duty ratio of } T_{B+}$$

PWM with bipolar voltage switching: (T_{A+}, T_{B-}) and (T_{A-}, T_{B+}) are treated as two switch pairs, when $v_{\text{control}} > v_{\text{tri}}$, T_{A+} and T_{B-} are turned on. Otherwise, T_{A-} and T_{B+} are turned on.

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d = (2D_1 - 1)V_d$$

PWM with unipolar voltage switching: a triangular waveform is compared with the control voltage v_{control} and $-v_{\text{control}}$ for determining the switching signals for leg A and leg B, respectively. Effective switching frequency of the output voltage waveform is doubled and the ripple is reduced @ same switching frequencies.

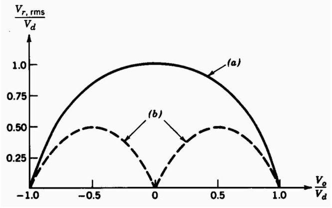

Figure 15: Unipolar vs bipolar

7.6 dc–dc converter comparison `pp. 195-196`

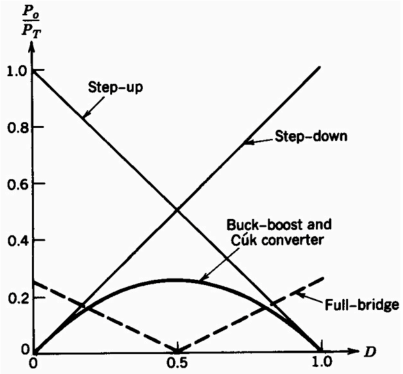

Figure 16: Switch utilization in dc–dc converters

Step-down, step-up, buck–boost, and Cúk converters are capable of transferring energy only in one direction. Full-bridge converter is capable of a bidirectional power flow. It is preferable to use either the step-down or the step-up converter from the switch utilization consideration.

8 Switch-mode dc-ac inverters: dc ↔ sinusoidal ac `pp. 200-248`

Task: produce a sinusoidal ac output whose magnitude and frequency can both be controlled.
Categories (3): Pulse-width-modulated inverters, square-wave inverters, single-phase inverters with voltage cancellation.

8.1 Pulse-width-modulated switching scheme `pp. 203-210`

$$m_a = \frac{\hat{V}_{\text{control}}}{\hat{V}_{\text{tri}}} \quad m_f = \frac{f_s}{f_1}$$

Harmonic spectrum: peak amplitude is m_a times $\frac{1}{2}V_d$, harmonics in the inverter output voltage appear as sidebands centered around the switching frequency and its multiples, the harmonic m_f should be an odd integer resulting in an odd symmetry and half-wave symmetry (only odd harmonics).
Small m_f ($m_f \leq 21$): synchronous PWM (m_f is an integer), asynchronous results in subharmonics.
Large m_f ($m_f > 21$): asynchronous PWM can be used because the amplitudes of the subharmonics are small.
Overmodulation: to increase further the amplitude of the fundamental frequency. many more harmonics, used with synchronous PWM.

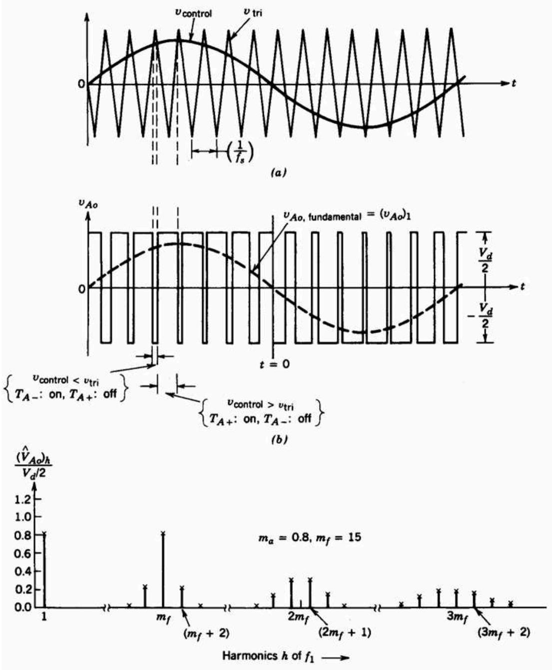

Figure 17: Pulse-width modulation

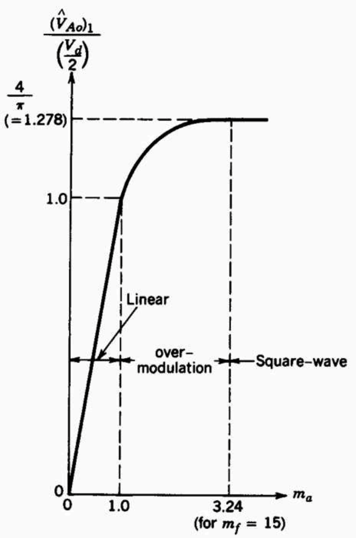

Figure 18: Voltage control by varying m_a

Square-wave switching scheme: each switch of the inverter leg is on for one half-cycle, only odd harmonics, state change only twice per cycle, many harmonics, unregulated output voltage.

8.2 Single-phase inverters `pp. 211-224`

dc-Side current i_d:

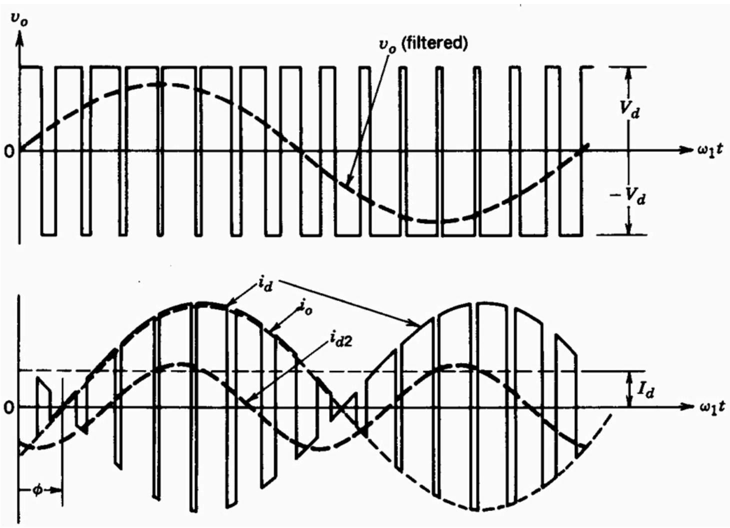

Figure 19: Bipolar dc-side current

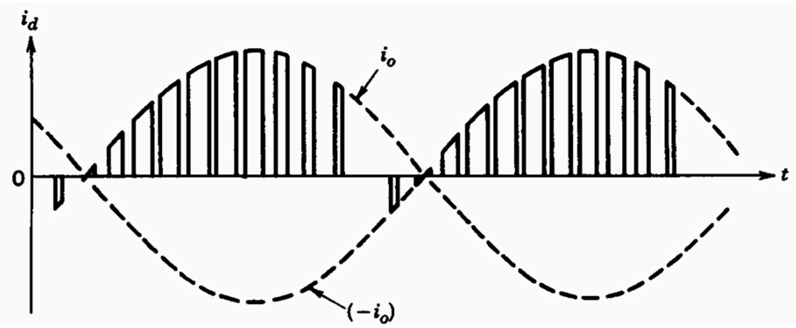

Figure 20: Unipolar dc-side current

8.3 Three-phase inverters `pp. 225-236`

8.4 Effect of blanking time on voltage in PWM inverters `pp. 236-239`

9 Resonant converters: zero-voltage and/or zero-current switchings `pp. 249-264`

Task: overcome the problems of switching stresses, switching power losses, and the EMI by turning on and turning off each of the converter switches when either the switch voltage or the switch current is zero.
Categories (4): load-resonant converters, resonant-switch converters, resonant-dc-link converters, high-frequency-link integral-half-cycle converters.

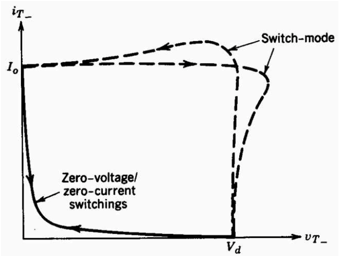

Figure 21: Zero-voltage-/zero-current-switching loci

9.1 Load-resonant converters `pp. 258-264`

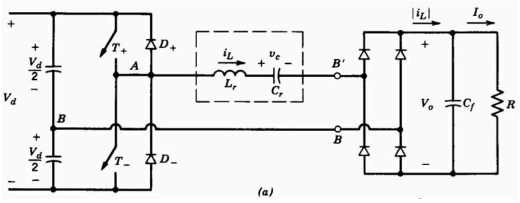

Figure 22: SLR dc–dc converter

Discontinuous-conduction mode with $w_s < \frac{1}{2}w_o$: the switches turn off naturally at zero current and at zero voltage, they turn on at zero current but not at zero voltage
Discontinuous-conduction mode with $\frac{1}{2}w_o < w_s < w_o$: the switches turn on at a finite current and at a finite voltage, they turn off at zero current and zero voltage.
Discontinuous-conduction mode with $w_s > w_o$: the switches turn on at zero current and zero voltage, they turn off a finite current.

10 Switching dc power supplies `pp. 301-310`

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About this document

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```
▷ latex keywords.tex
▷ dvips -Ppdf -tA4 -G0 keywords.dvi
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```

Written by: *Jonathan Goldwasser* — Last revision: *12th June 2005*